Central Coast Climate Science Education
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The Basic Physics and Math of the Passage of Infrared Radiation
Through the Earth's Atmosphere, and the Greenhouse Effect

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Several years ago, I wrote a post on "How the greenhouse effect works". But, from time to time, I still am sent copies of articles written by individuals who purport to show that there is no such thing as the greenhouse effect. Or, as a variant on this theme, that the extremely high temperature on the surface of Venus is "simply due to the high atmospheric pressure" and not the greenhouse effect.

These assertions are dressed up in scientificky-sounding jargon and can sound seductively authoritative, but they are, to be blunt about it, total nonsense.

I want to first suggest to readers that they watch a video which I generated and described in the post I referred to above, if they have not already seen it. It is a simulation of how infrared radiation works its way through the atmosphere and in the process illustrates the greenhouse effect. It uses only the most basic and well-understood concepts of physics, all of which are explained in the video. See here to view it: https://www.youtube.com/watch?v=9DaohdBhbfQ
And see here for the accompanying text: https://tinyurl.com/wnf86t6w .

However, I thought it would be of interest to teachers and students at the college level, (and in fact advanced high school students taking AP Calculus and AP Physics-2) to show how one quantitatively calculates the transport of radiation through the Earth's atmosphere. In what follows, the basic physical concepts are explained, and, although I have used a little bit of basic calculus in showing what is involved in the calculations, those without a knowledge of calculus should still be able to follow the logic of the formulation.

First, some definitions and an explanation/review of some basic concepts of physics for those for whom this material is new or who are a little rusty.
In an Appendix I have put a glossary of these same concepts and definitions.

Some basic concepts of modern physics and some definitions

Electromagnetic Radiation: Wave or particle? "Photons"

The great genius Isaac Newton believed that light consisted of a stream of particles (the "corpuscular" theory of light), but others, including the Dutch physicist Huygens and the French physicist Fresnel, proposed instead that light should be understood as a wave phenomenon. So great was Newton's prestige, however, that it was not until an experiment performed at the beginning of the 19th century by Thomas Young—the famous "double slit" experiment[1]—that the wave interpretation gained widespread acceptance.

There matters stood until late in the 19th century when Hertz discovered the photoelectric effect[2], but it remained for Einstein in 1905 to explain its puzzling features. He did so by proposing that light could, under phenomena such as the photoelectric effect, be best understood as consisting of particles or "quanta" of energy, now called **photons.** The amount of energy carried by one of these photons is given by the expression

$$E_{photon} = h*\nu$$

In this formula the Greek letter ν represents the frequency of the radiation[3] when it is thought of as a wave, and **h** is a physical constant—Planck's constant---the same constant Max Planck invoked to formulate a consistent expression for black-body radiation, or "Planck radiation", discussed below. The apparent paradox that radiation could sometimes be considered a wave and sometimes a particle, depending upon the type of observation or measurement, was resolved with the development of quantum physics.

When considering how radiation interacts with atoms and molecules, it is, as in the case of the photoelectric effect, more appropriate to think of radiation in terms of its photon aspect, rather than its wave aspect.

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[1] https://www.olympus-lifescience.com/en/microscope-resource/primer/java/doubleslitwavefronts/

[2] See here for a discussion of the photoelectric effect: https://simple.wikipedia.org/wiki/Photoelectric_effect

[3] **Infrared radiation is part of the more general case of "electromagnetic radiation") for which visible light is also** just that portion of electromagnetic radiation which has frequencies such that we can see that radiation with our eyes. Lower frequencies (or longer wavelengths) of electromagnetic radiation comprise the infrared part of the "electromagnetic spectrum." We will mostly be concerned with infrared radiation. The term 'radiation' should not be confused with the "radiation" produced by radioactive atoms, for which, see:
https://en.wikipedia.org/wiki/Radioactive_decay

"Quantized energy levels"

When being introduced to the nature of atoms as consisting of a tiny dense positively charged nucleus, with small negatively charged electrons whizzing around it in orbits, teachers sometimes use the analogy of planets moving in their orbits around the sun. But there is a fundamental difference. The size and shape of such a planetary orbit, and therefore its total energy—kinetic energy and gravitational potential energy—is not set by any fundamental physical law. The earth's orbit could have been a little larger or a little smaller and a little less or a little more circular.

The situation is totally different at the very small sizes of atoms and molecules. The laws of quantum physics rule at these sizes and restrict the energies of the electrons in their orbits for both atoms and molecules to very definite, well-defined values of energy. In the case of molecules, the possible energies with which molecules can rotate and vibrate are similarly only strictly permitted energies.

Molecules in some particular high energy vibration and/or rotation 'state', say E_2, can drop down to a lower energy state, say E_1, by spontaneously emitting a photon in a random direction whose energy is just the difference $E_2 - E_1$. Conversely, if a photon comes along with just this same energy difference it can be absorbed by a molecule in its low energy state, with this energy used to make the transition from the low to high energy state.

Absorption cross section

You can think of a molecule as having a certain area which serves as a 'target.' The area of this target, usually denoted with the lower case Greek letter sigma σ, is for photons having just the right amount of energy to increase the energy of the molecule from one of several low energy states E_1 to one of its many higher energy states E2. This higher energy state will have an increased amount of vibrational and rotational energy, as explained above. This area, σ, will depend on, and generally varies greatly for, every combination of E_2 and E_1 and thus for the energy of the particular photon, which must be equal to $E_{photon} = E_2 - E_1$.

Intensity

(For which astronomers usually use the symbol **I**): Imagine a beam of radiation (or if you like, a stream of photons) and a tiny surface perpendicular to that stream

having cross section ΔA: I is defined such that **I*ΔA** measures the rate at which energy crosses that surface (or we could equally say the number of photons carrying energy $E_{photon} = h\nu$) across that surface every second.

The basic equations of the passage of radiation through the atmosphere

Now imagine a long skinny tube having an area of **ΔA** cm^2 and extending from the earth's surface to the "top" of the atmosphere (i.e. high enough so that scarcely any atmosphere remains above it.) Suppose we put a bunch of CO_2 molecules in the tube so that the density of molecules at one point in the tube is **n** molecules per cubic centimeter. This **n** refers to the number in a lower energy state and below I will make this explicit by writing instead **n(E1)**

Now consider further a tiny slice of that tube having thickness **Δs** cm. The volume of that little slice will then be **$\Delta A \, \Delta s$**, so the number of molecules in that slice will be **n$\Delta A \, \Delta s$**. Since each of these molecules can be thought of as having a little target area of **σ,** the total area in this little slab covered by these little targets is **n$\sigma \Delta A \, \Delta s$** square centimeters. We can choose the little slab thickness to be so small that the area **n$\sigma \Delta A \, \Delta s$)** is *much* less than **ΔA,** the tube cross section itself, *i.e.* so none of the targets will overlap. Thus, the *fraction* of the area of the tube in this tiny slab covered by these targets is **nσ*ΔA*Δs/ΔA = nσ*Δs.**

This means that if we shine a beam of photons of intensity **I** at the bottom of this little slab it will be weakened by just this tiny fraction, or in symbols:
$$\Delta I = - I \, n\sigma(\Delta s)$$

The minus sign goes there because as we move through the tiny slab in the direction of the stream of photons, what comes out the other end is *less* than what went in. Now divide both sides of this equation by the tiny slab thickness. It was the genius of the inventors of differential calculus to make rigorous the notion of the "*limit as Δs goes to zero*" and what we get is a simple *differential equation*:

Equation (1)   **dI/ds = - I * n(s) * σ**

I have written the density as n(s) , showing that the density of CO_2 molecules is not usually constant but varies as we move along a distance s, because the density of the CO_2 molecules will decrease as we move upwards through the atmosphere. For the moment, though, suppose **n** is constant. Then the solution of equation (1) is *the law of exponential decay:*

Equation (2) $I(s) = I_o * e^{(-n\sigma s)}$ where I_o is the intensity at some starting point, say the surface of the earth, and s would be the altitude. In the video explaining the greenhouse effect I have introduced the idea of exponential decay in a different way using the analogy of 'negative compound interest.' We can think of this law as being the *probability* of a single photon traveling a certain distance, but since the sum of all probabilities must be one, we have to 'normalize' the probability law. This results in the probability of a photon traveling a distance between s and s + ds[4] as

$$p(s)*ds = n*\sigma* e^{(-n\sigma s)} * ds$$

The *average*, or "mean" distance the photon travels is calculated (using calculus) by weighting the probability by **s** that the photon travels a distance between **s** and **s+ds** before being absorbed**.**
This average distance is equal to **1/ nσ** and is called the "*mean free path*", or **mfp**. So, if we measure the distance from one point to another and that distance is **s** centimeters long, then the *number of 'mean free paths'* it has traveled is the distance **s** divided by the mfp which is equal to **nσs.**

Optical distance **or '*optical depth'***

We can generalize this to the case where **n** is not a constant with **s**, but varies with altitude, but we can still calculate the number of mean free paths (numerically, if necessary) by adding up the density of molecules in tiny slabs of thickness Δ**s** between any two points along some path. The result is still the number of mean free paths.

It is easier to calculate the passage of radiation through the atmosphere *using the number of mean free paths*, rather than the physical distance, and this "distance", (for which the Greek letter τ is conventionally used) is called the 'optical distance.' This is because the photons don't care about the actual distance in kilometers they travel, but only the number of, for example, CO_2 molecules they encounter along the way.

To calculate the emergent radiation as it is observed by satellites, we consider photons traveling upwards but we will measure the optical distance from the *top of*

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[4] The probability that the photon travels *exactly* a distance **s** is zero, so we consider a tiny little slab of thickness **ds** and the probability that the photon travels a distance somewhere in the interval **s** and **s + ds** is finite.

the atmosphere (TOA) <u>*down*</u> to some point in the atmosphere (or all the way down to the surface.) Measuring it in this way is called the *optical depth* of that point. From this point of view, we could write equation (2) as representing the radiation starting from some point in the atmosphere (or at the surface) which makes it all the way to the top of the atmosphere as:

Equation (3) $\mathbf{I(toa) = I_o * exp(-\tau)}$ (as in equation 2)

[From now on I will write "e" raised to some power as exp( ).]

However, it is *extremely important* to remember that the *optical depth* down to a given actual physical point depends *very strongly* on which energy photon (or frequency[5] of radiation) we are dealing with, since over just the short span of frequencies over which we see a 'dip' from the absorbed radiation from, say CO_2 , (see figure 1 below) the value of the cross section changes by a very large amount.

If we were to calculate the intensity of radiation leaving the top of the atmosphere and use equation (3) to do so, where I_0 is the intensity of the black body radiation leaving the Earth's surface and τ is the optical depth through the entire atmosphere for the particular photon energy we are considering, would we get the correct answer? Definitely not! The reason is that some radiation is emitted in the upward direction *by the atmosphere itself*, so we have to consider that in a quantitative way:

<u>The *emission coefficient* and *"local thermodynamical equilibrium"*</u>

We have defined the absorption coefficient, σ, above. We also suppose we are given the number of molecules per cubic centimeter in the lower energy state, $n(E1)$. It is only those molecules that can absorb a photon having energy $E2 – E1$. When such an absorption takes place, that particular molecule will then find itself in the upper energy state E2.

Molecules in this upper state can then emit another photon of the energy difference E2-E1 in a random direction.

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[5] Equally well, in terms of the wave aspect of radiation, one can use the wavelength, λ, of the radiation, which is related to the frequency by the formula $\lambda = c/\nu$ where c is the speed of all electromagnetic waves, including light. For a discussion of the wave nature of electromagnetic radiation and the relation between wavelength and frequency see, for example: https://www.elprocus.com/relationship-between-wavelength-and-frequency/

We <u>define</u> the emission coefficient, ε (lower case Greek epsilon) such that if there are n(E2) molecules per cubic centimeter in this upper energy state, then the rate at which these emitted photons occur is given by n(E2)* ε. *The quantities σ and ε are closely related* since they are sort of just the opposites of each other, and their ratio just depends in a simple way on the frequency of the photon we are talking about.

We thus need to correct equation (1) to take account of the fact that *both* absorption *and* emission are taking place, and this corrected equation (1) then looks like:

Equation (4)   dI/ds = - I * n(E1) * σ + n(E2)* ε

We do a little bit of manipulating of equation (4) by dividing both sides by **n(E1)* σ** Then, recalling what we meant by, and defined optical distance to be, we can just as well think of a little slice of optical distance
dτ = n(E1)* σ*ds instead of a little slice of physical distance **ds.** This makes equation (4) now look like this:

Equation (5)   dI/ dτ = -I + (ε/σ) * n(E2)/n(E1)

So, for a given frequency of photons, if we know the total number of, say, CO_2 molecules in the atmosphere, as well as the cross section for the photon energy in question, then the remaining piece of information that we need is the ratio of n(E2)/n(E1) at every point in the atmosphere, since the ratio (ε/σ) is a known simple function of frequency. *The ratio* n(E2)/n(E1) *depends upon which processes are most important in moving a molecule between the lower and upper energy states E1 and E2.*

The absorption and emission of a photon is certainly one way.[6] But another way is for another passing molecule that has enough kinetic energy to "kick" a CO_2 molecule from state E1 to E2 (and not necessarily, or even usually, another CO_2 molecule, but more often a passing nitrogen or oxygen molecule, since they are so much more abundant in the Earth's atmosphere.) This process is called *"collisional excitation."* The opposite process will also occur: For a CO_2 molecule in its upper energy state E2, the energy difference E2-E1 is communicated to a passing molecule and gives that molecule an increase in its kinetic energy, leaving the CO_2

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[6] In addition to the "spontaneous" emission of photons by a molecule in some upper energy state, emission can be "stimulated" by passing electromagnetic radiation. Under the right circumstances this stimulated emission is the essence of the laser phenomenon.

molecule back in state E1. This reverse process is called (not surprisingly) "collisional **de**-excitation."

Here is a crucial point: In the Earth's atmosphere these *collisional* process occur *far* more frequently than the rate of absorption and emission of a photon, though of course these two photon processes *do* occur, since that is what governs the energy balance, hence temperature, at every layer of the atmosphere.[7] Under these circumstances, a result, established way back in 1868 by the famous physicist Ludwig Boltzmann, is that the ratio n(E2)/n(E1) depends only on the temperature through the formula

$$n(E2)/n(E1)= \exp[\,-\,(E2\text{-}E1)/kT]$$

where k is "Boltzmann's constant" and T is the temperature in degrees Kelvin.[8] *A second crucial point is this:* When this condition holds in a volume of gas, the quantity $(\varepsilon/\sigma) * n(E2)/n(E1)$ *is just the Planck function*[9]*, B(v,T) even though* the actual radiation in which this volume of gas is bathed, is not the Planck function.

This condition is called the condition of "local thermodynamic equilibrium" (**LTE**) *and makes the solution of equation 5 far simpler* than the situation which one sometimes encounters in the atmospheres of some stars, especially if we know in advance how the temperature and density vary with height in the earth's atmosphere, which is usually the case.

 In order for the radiation to actually be closely equal to the Planck radiation we would need to have a volume of gas be at nearly the same temperature over distances such that the *optical* distance over this region is big enough that few photons from different temperature regions enter the region, and few photons within this region escape it. Nevertheless, under the conditions of LTE, equation (5) now takes the form:

Equation (6) $dI_v / d\tau = -I_v + B(v,T)$

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[7] Though usually, convective transfer of heat also plays a role in fixing the temperature at each point in the atmosphere.

[8] https://en.wikipedia.org/wiki/Boltzmann_distribution

[9] The formula for the Planck function, also called "Black body radiation" is
  $B(v,T) = (2hv^3/c^2) * 1/(\exp(hv/kT) -1)$

(I have put the ν as a subscript on the I quantities and in the Planck function *B(ν,T)* as a reminder that we are dealing with photons of a particular frequency. I have also glossed over the notion of a very small cone of *solid angle* which the I and B refer to; just accept the fact that in equation (6) the amount of Planck function emission is that fraction headed in the same direction as the beam of radiation we are following.)

From the standpoint of pure mathematics, equation (6) is a first order, linear, nonhomogeneous, differential equation, since T is supposed to be a known function of τ. If you have taken a course in differential equations, you might recall that since the homogeneous equation solution to equation (6) is known, you can find the nonhomogeneous solution using the technique of an "integrating factor." But we can write down the solution *simply on the basis of understanding what is happening to the stream of photons heading to the top of the atmosphere.* I am going to use the notation of integral calculus to write this down and then explain it, using the concepts discussed above:

Equation (7) I_ν (emerging from top of atmosphere) =

$$\int_0^{\tau\, total} B\big((T(\tau)\big) * exp(-\tau)d\tau$$

The physical meaning of this expression is this: We imaging a large number of layers of the atmosphere each of tiny (optical) thickness $d\tau$. At each optical distance τ, measured from the top down, (with the layers going all the way to the total optical distance at the surface of the earth) a particular layer located at optical distance τ from the top emits an amount of radiation given by the Planck function at the temperature of that layer times the tiny thickness of that layer, $d\tau$. But we know that on its way to the top of the atmosphere those particular photons are reduced exponentially by **exp(-τ).**

I have not included the additional contribution that the emission *directly* escaping from the surface of the earth makes to the radiation emerging at the top of the atmosphere, which is

Equation (7a) $I_{(toa)} = I_{(surface)} * exp(-\tau_{total})$

This is obviously not important unless the entire atmosphere is nearly transparent at the frequency under consideration since otherwise the exp term chokes off the radiation from the surface.

The symbol ∫ used in integral calculus is just a rigorous way of adding the contributions each of these tiny layers makes to the radiation escaping from the top of the atmosphere.

Consider a particular frequency of those photons which can be absorbed by a CO_2 molecule and suppose that for this frequency the total optical thickness is 10, i.e. there are 10 mean free paths. Even though it is warmer down near the surface so the Planck function at that temperature produces more radiation than cooler layers, the radiation from that particular layer is reduced by exp(-10) which is 0.000045!

So, those low layers simply make no contribution to what comes out at the top. The photons from the last few layers, very near the top, say about 0.1 mean free path from the top, can easily escape, but there aren't very many of these layers so they also don't contribute very much compared to all the other layers not too far down.

If one makes the simple mathematical assumption that B(τ) varies in a *linear fashion* over the last few mean free paths from the top, (it doesn't really matter what happens deeper down because of the exponential decay) then it turns out that the radiation which emerges *is equal to that which would be produced if there were an actual black body radiating at the temperature of a layer exactly one mean free path from the top of the atmosphere.* This same results holds approximately without this assumption unless the temperature does something really crazy in these last few mean free paths.

This last result provides an intuitive way of understanding the character of the radiation field emerging at the top of the atmosphere. Figure 1 is a space-based observation of this radiation[10].

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[10] Note that the top scale in wavelengths is not uniform. The bottom scale is uniform and is measured in 'inverse centimeters', i.e., $1/\lambda$ where λ is measured in centimeters and is proportional to the frequency, thus photon energy. See footnote 5 for a discussion of the relation between wavelength and frequency.
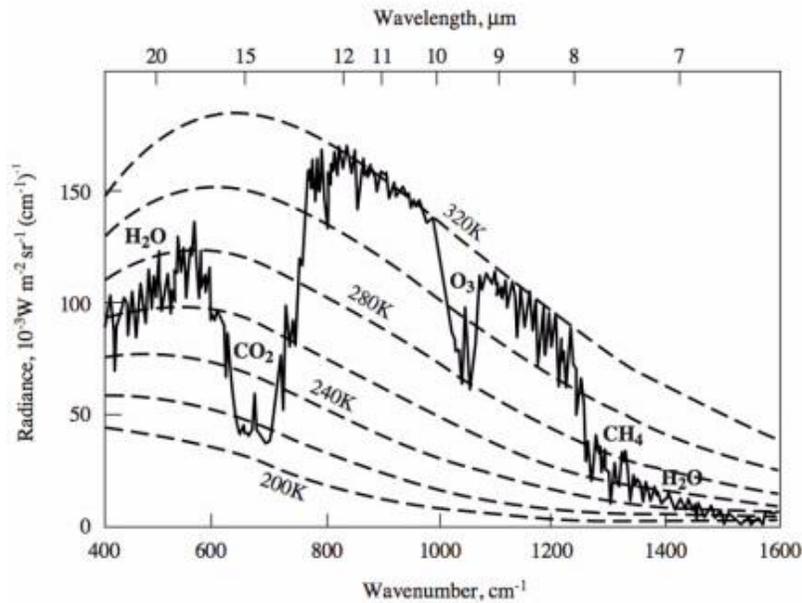
Credit: Data from R. A. Hanel, et al., J. Geophys. Res., 1972, 77, 2829-2841

Figure 1. The observed infrared radiation emerging from the top of the atmosphere as measured from space over a hot surface of North Africa. See the text for the interpretation of the various dotted curves labeled with various temperatures. The dotted lines are Planck function curves labeled for various temperatures.

But in so doing it is crucial to realize that the cross section σ for absorption varies hugely with frequency (or wavenumber) as one moves across the region where CO_2 can absorb radiation. At the point on the graph near wavenumber 750 or so, where the observed radiation crosses the Planck curve marked 280K, one mean free path for those photons corresponds to a layer in the atmosphere where the temperature is about 280K, based on the discussion above. As we move toward wavenumbers where the cross section is much larger, the layer at one mean free path is much higher up in the atmosphere, so that at wave number about 700 the "one mean free path layer" has a temperature is lower, about 215K or so.

We can even understand the little "spike" right in the center of the CO_2 trough at a about wavenumber 680. Here, the cross section is so large that the one mean free path layer occurs up in the *stratosphere* where the temperature has *risen* a bit and so the higher temperature and corresponding Planck function is a little higher, hence the spike. Where the cross section drops off to zero, photons from the surface itself can escape and we see a Planck function associated with the surface temperature whose intensity is then given by the surface value of the Planck

11

function since (see equation 7a) the total optical depth is almost zero and exp(-0) =1.

One sentence **summary**: *The intensity of the emerging radiation from the top of the atmosphere at a given frequency may be thought of as coming from a layer emitting black body radiation at the temperature associated with that layer of the atmosphere located about one mean free path from the top of the atmosphere.*

"Down welling" radiation and it's relevance to global warming

We can equally well calculate the amount of "down welling" radiation as measured from the ground looking up, but instead of optical distance in equation (7) being measured from the top down, it is measured from the bottom up. In this case, the intensity of this down welling radiation will be approximately equal to the Planck function at an optical depth of one *measured from the surface upward,* where the Planck function has a higher value because of the warmer temperature of the atmosphere near the ground, compared to the temperature near the top of the atmosphere. *This downwelling radiation presently contributes about twice as much energy towards surface warming as direct warming of the earth's surface by absorption of sunlight!*

It is the increase in this downwelling radiation which causes the increase in the earth's temperature. As more CO_2 is continually added to the atmosphere, the optical thickness of the atmosphere for each frequency in the region around the CO_2 dip around wave number 700 increases (see Figure 1.) For those frequencies where the optical thickness is already very large this further increase in CO_2 will result in little increased downwelling radiation—most of the energy emitted from the Earth's surface at those frequencies is already being returned, a situation referred to as "saturation". But even though the absorption cross sections decrease away from the center of the CO_2 dip, eventually those frequencies too will reach optical depths greater than one with increasing amounts of CO_2 and will contribute to increased down welling radiation. This results in increased warming of the earth. *Further increased temperatures will continue as long as we continue to pour more greenhouse gases into the atmosphere.*

(Failure to appreciate the role of these frequencies away from the middle of the CO_2 dip as their optical depths become appreciable have resulted in erroneous statements by some who argue that "saturation" means adding more CO_2 to the atmosphere will not result in more global warming.)

Concluding remarks and a valuable additional resource

The first remark has to do with the presence of convective motions which move energy 'in bulk' vertically through the atmosphere. This additional transport is so efficient that to a high degree of accuracy the atmosphere follows what is known as an 'adiabatic gradient' (which can be either 'wet' or 'dry') But equation (7) is still entirely valid. *Even in the presence of convection, the majority of the energy flowing upwards through the earth's atmosphere and escaping into space is by means of radiation, not convection.*

A second remark is that when atmospheric climate scientists specializing in this field carry out these calculations, they do so by including an enormous number of individual transitions, i.e., photon energies corresponding to the energy difference E2 - E1, for a huge number of energy upper and lower energy states E2 and E1, each with a particular absorption cross section. This is done not only for carbon dioxide but also for the many other molecules which absorb significant infrared radiation, especially water vapor. These 'line-by-line' calculations require large amounts of time on supercomputers to carry out.

A third remark is that I have, not surprisingly, glossed over some subtleties in this discussion. There is never a single *exact* energy for a given transition between a lower and higher energy state, since both these energy states are blurred ("broadened") by various processes, especially being disturbed by passing molecules. This leads to a finite width and a variation of the cross section over the narrow energy region of a given transition. Other complications arise when transitions between two different kind of molecules overlap in energy. Some of these subtleties still involve uncertainties, but research has shown that these are much too small to affect the overall understanding of the magnitude of the greenhouse effect.

As a justification for this last remark, Figure 2 is an example of such a "line by line" calculation. There is excellent agreement with Figure 1.
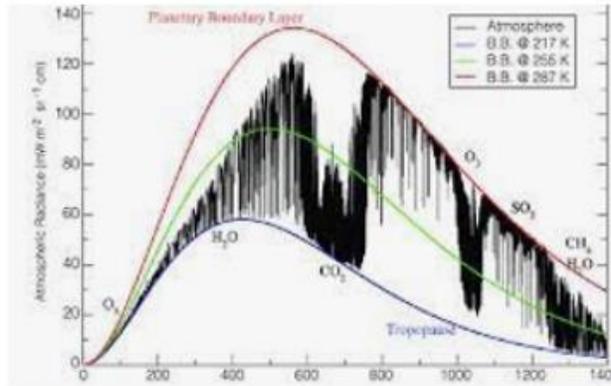
Figure 2 An example of a detailed "line by line" calculation of the radiation emerging from the top of the atmosphere. The agreement between the observed radiation in Figure 1 is very good, though the detail shown in this calculation is much greater than is able to be observed from space.

An excellent supplementary and highly readable resource

For those interested in an account of the greenhouse effect and radiative transfer, not just in the earth's atmosphere, but in the atmospheres of Mars and Venus, I ***highly recommend*** an article by a leading expert in this area, Dr. Raymond Pierrehumbert. It was published in 2011 in *Physics Today,* whose archives are now publicly available. Rather than attach it here, I have put it as a separate resource in my website: Resources→ For Students and Teachers.

I am greatly indebted to Dr. Beverly T. Lynds for a careful reading of early drafts of this essay and for many helpful suggestions for improving it.

Appendix: Glossary of Concepts and Definitions

Absorption cross section: The effective area of a molecule presented to a stream of photons of energy E2 – E1 to intercept the *photon* and raise it from low *energy state* E1 to a high energy state E2.

Black body radiation: The *intensity* of *electromagnetic radiation* over all frequencies inside an idealized enclosure with walls all at exactly the same temperature, and that fully absorb and emit radiation at all frequencies. This radiation is also called Planck radiation.

Electromagnetic radiation: When thought of as a wave phenomenon, it is a vibration of electric and magnetic fields which travel through space at the speed of light. Depending upon the wavelength (or alternatively, the frequency of the wave) it manifests itself as gamma rays, x-rays, ultraviolet, visible light, infrared, and radio waves as one considers extremely short to very long wavelengths.

Emission coefficient: The rate at which a collection of n(E2) molecules per cubic centimeter (in energy state E2) spontaneously emit electromagnetic radiation of energy E2 - E1, as they transition to lower energy state E1.
There is a close relation between the emission coefficient and the absorption coefficient.

Energy levels: See quantized energy levels

Frequency: See wavelength for an explanation and the relation between wavelength and frequency.

Infrared radiation: Electromagnetic radiation having wavelengths roughly 2 to 8 times that of visible light ("near infrared") or 8-100 times that of visible light ("far or "thermal" infrared). The earth's surface emits thermal infrared radiation.

Intensity: A measure of the rate at which electromagnetic radiation flowing in a particular direction transports energy across a small imaginary surface perpendicular to that direction.

Local thermodynamic equilibrium: ("LTE") A situation in which collisions between molecules occur with sufficient frequency that the distribution of the speeds of the molecules and their distribution over all their energy states is virtually identical to that which occurs in true thermodynamic equilibrium at some temperature, T. Under these conditions, the quantity $(\varepsilon/\sigma) * n(E2)/n(E1)$ in equation 5 is just the Planck function.

Optical distance/Optical depth: Imagine a tube ("column") of one square centimeter cross section in which there are N molecules (i.e., a "column density of N molecules per square centimeter,) each with absorption cross section σ square centimeters, between two points. Then the *optical distance* between these two points, τ, is $N\sigma$. (If the molecules have uniform volume density, n along the way, then N = ns if s is the physical distance separating them, in which

case $\tau = n\sigma s$.) The stream of *photons* starting at one of these points is decreased by $\exp(-\tau)$ by the time it reaches the 2nd point.
Optical depth generally refers to the optical distance measured downward from the top of the atmosphere down to some point in the atmosphere or the earth's surface.

Photon: A "particle" of electromagnetic radiation carrying an amount of energy given by $E = h\nu$, where ν is the frequency when the electromagnetic radiation is interpreted as a wave phenomenon. Electromagnetic radiation is manifested in its particle aspect when absorbed or emitted by molecules, or when it hits a photocell or solar panel.

Planck radiation/Planck Function: Planck radiation is another name for blackbody radiation (q.v.), and the intensity of that radiation is given by the formula in footnote 9.

Quantized Energy Levels: The rules of quantum physics dictate that the amount of energy involved in the motions of electrons about the nuclei of atoms or molecules and the energy of vibration and rotation of molecules is limited to precise discrete "quantized" values. These values can be experimentally measured, or in some cases calculated.

Wavelength and Frequency: See footnote 5 for a discussion, though I do not find the explanation for the relation between them very clear. Here is mine:
Picture the wave as a series of peaks and troughs moving past you (as in waves of water.) Let the symbol λ denote the distance between successive peaks—i.e., the wavelength of the waves. Suppose the waves are traveling at some speed "c" and at a fixed point you measure the time it takes N of these peaks to pass you. The distance the last peak has to travel before it passes you is $\lambda*N$, and since the wave is traveling at speed "c" the time it takes for all these peaks to pass you is distance/speed, so t (the time it takes for all the waves to pass you) is $t = \lambda*N/c$. By frequency, for which the Greek letter ν is used, we mean the number of peaks that pass by us every second. So, for example, if 100 peaks ("N") pass by us in 25 seconds ("t") we would say the frequency of the wave was $100/25 = 4$ cycles per second. Or, in the more general symbols we have used above: $\nu = N/t$. Rearranging the expression above, $t = \lambda*N/c$, using simple basic algebra:
   $N/t = \nu = c/\lambda$ which is the fundamental relation between frequency, wavelength, and the speed of the wave, which in our case is the speed of all electromagnetic waves, including light.